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Spin electromagnetic field and dipole: A localized and quantized field

FENG ZhengHe

National Laboratory of Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

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This paper investigates the structure of a spin electromagnetic (EM) field and its various physical properties. A spin EM field is an intrinsic mode of free space, which satisfies the spin equations derived from Maxwell equations. A spin mode has two basic properties: the spin along its axis and the localization of electromagnetic field. The source and EM structure of this electric and magnetic mode are described in detail in this paper. The distributed charge and current of a spin mode can be integrated to obtain the electric and magnetic moment, and they can be treated as the electromagnetic dipole. A spin mode possesses both wave and particle properties: a wave number, angular frequency and characteristic speed are its wave parameters; an intrinsic radius, energy and angular momentum are its dynamic parameters. The former is analogous to an EM resonant mode; the latter is similar to the behavior of a particle with intrinsic spin. There are two kinds of electric modes present, one can be expressed through a pair of charges, and the other can be expressed by a magnetic current. They both have the same electric moment, but have different divergence properties.

spin electromagnetic field, spin equation, distribution source, dipole, moment, intrinsic radius, quantized, Plank constant

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The concept of a dipole is fundamental in electrodynamics and quantum mechanics. Before Maxwell's theory electromagnetism was well established, Lorentz described the properties of dielectrics in terms of an electric dipole, namely, representing an electric dipole using a pair of positive and negative charges separated by distance l. Therefore, $\overline{p}=q\overline{l}$ is the electric dipole moment [1,2]. Weber explained the magnetic characteristics of matter in terms of a magnetic dipole [3], namely, representing a magnetic dipole using a small current loop of area, S. Therefore, $\overline{m}=I\overline{S}$ is the magnetic dipole moment [4]. Lorentz explained the dielectric dispersion mechanism by introducing the time factor, $\mathrm{e}^{i\omega t}$, into the dipole moment. Therefore, he used $\overline{p}\mathrm{e}^{i\omega t}$ to describe the dipole's harmonic time variation [5]. To explain the radiation characteristics of dipoles, Hertz introduced a retarded potential using a phasor. By adding factor of e^{-ikr} or $\mathrm{e}^{-i\omega R/c}$, the harmonic varying dipole could be given radiative characteristics [6–8]. In quantum mechanics, dipole is used in different layers of the structure of matter. It is used in objects such as the electron, atom, nucleus and molecule to analyze the interactions among the EM field, waves and matter.

The dipole's basic EM structure can be calculated using the superposition theorem [9]. The electric field distribution of an electric dipole composed of charges is depicted in Figure 1(a), and the magnetic field distribution of a magnetic dipole, which is generated by a small current loop, is shown in Figure 1(b).

Spin is a fundamental property of the microscopic world. Microscopic particles, such as electrons, protons and neutrons, can be treated as spinning spherical charges. They have a fixed magnetic moment, angle momentum and spin quantum number. The spin of the electron plays an important role

Email: fzh-dee@mail.tsinghua.edu.cn

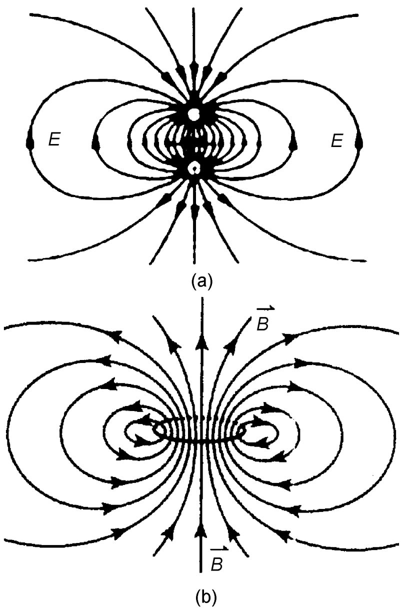

Figure 1 Electric dipole (a) and magnetic dipole (b).

in quantum physics and solid state physics. Using concept of spin, we can model the interaction between electromagnetic field and matter on many different levels; including that of the electron, atom, nucleus, molecule and crystal lattice. Recently, spintronics has been rapidly developed [10–18]. The physical manifestation of spin is angular momentum, which has a fixed relationship with the dipole moment.

The electromagnetic field is also a physical entity. For spin in particular, there should be an intrinsic spin for some electromagnetic fields. In addition, a fundamental formula of electromagnetic theory states that two curl equations guide the electric and magnetic fields. This suggests the existence of a spin electromagnetic field.

This paper proposed the concept of a spin electromagnetic field and derived the spin equation from Maxwell's equations. Treating a distributed source as a part of an intrinsic mode, we can derive the governing equations from the spin equations. The solution of these equations is the spin electromagnetic field, which includes an electric and magnetic mode. These modes can be treated as the interior EM structure of dipoles and spin particles. In addition to the expression of electric mode using a pair of charges, the expression derived in this paper also uses a magnetic current. These two kinds of electric spin modes expression have the same electric moment. However, they possess different energy and divergence properties, because they have different sources.

The EM structure, wave properties, dipole properties and dynamic properties of the spin EM modes will be discussed in this paper in detail.

1 Spin electromagnetic field

1.1 Wave equation

Maxwell's equations in free space are

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}, \tag{1}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}, \tag{2}$$

$$\nabla \cdot \overline{E} = \frac{\rho}{\varepsilon}, \tag{3}$$

$$\nabla \cdot \overline{B} = 0, \tag{4}$$

where $\overline{E}$ and $\overline{B}$ indicate the electric and magnetic fields, respectively. ρ and $\overline{J}$ are the volume charge density and surface current density, respectively. By introducing the vector potential $\overline{A}$ and the scalar potential ϕ, and defining

$$\overline{E} = -\frac{\partial \overline{A}}{\partial t} - \nabla \phi, \tag{5}$$

$$\overline{B} = \nabla \times \overline{A}, \tag{6}$$

we obtain the wave equations:

$$\nabla^2 \phi - \varepsilon\mu \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\varepsilon}, \tag{7}$$

$$\nabla^2 \overline{A} - \varepsilon\mu \frac{\partial^2 \overline{A}}{\partial t^2} = -\mu \overline{J}, \tag{8}$$

under the Lorenz gauge:

$$\nabla \cdot \overline{A} + \varepsilon\mu \frac{\partial \phi}{\partial t} = 0. \tag{9}$$

Eqs. (7) and (8) are a group of inhomogeneous wave equations with charge and current sources. ε and μ in the above equations denote the medium parameters in free space. The time variable can be separated from wave equation. Assuming the magnitude of the electromagnetic field has harmonic time variation, we can obtain the Helmholtz equation:

$$\nabla^2 \phi + k_0^2 \phi = -\frac{\rho}{\varepsilon}, \tag{10}$$

$$\nabla^2 \overline{A} + k_0^2 \overline{A} = -\mu \overline{J}. \tag{11}$$

Traditionally, we either introduce a δ-function on the right-hand side to calculate retarded potential or a Green's function [19]. Alternatively, we could set the right-hand side to zero and introduce boundary conditions to solve the eigen equations.

1.2 Spin equation

The key hypothesis in the separation of time and space variables described in previous section is that the EM field varies harmonically in time. The time derivative of the field is $\frac{\mathrm{d}}{\mathrm{d}t}=i\omega$, which has an imaginary frequency. This results in the EM wave propagating along a straight line.

However, this is not the only choice for three-dimensional EM-field analysis. Now let us consider a different method.

As depicted in Figure 2, assume the EM field is rotating in φ direction about the z-axis. In an eigenmode, the EM field maintains its structure during spinning. It can be treated as a spherical spinning object about the axis. Every point on the object rotates at the same angular velocity ω_0. The angular velocity is $\omega_0=\frac{\mathrm{d}\varphi}{\mathrm{d}t}$, which is equivalent to angular frequency of the EM field. However it is a real number in the spin mode.

The time derivative of ϕ is $\frac{\mathrm{d}\phi(\overline{r},t)}{\mathrm{d}t}=\omega_0\phi(\overline{r},t)$, where ω_0 is spin frequency. Substituting this into eq. (7), and letting

$$k_0=\omega_0\sqrt{\varepsilon_s\mu_s}, \tag{12}$$

we obtain

$$\nabla^2\phi-k_0^2\phi=-\frac{\rho}{\varepsilon_s}. \tag{13}$$

The same conditions are also applied to the vector potential function $\overline{A}(\overline{r},t)$ in eq. (8). Then we have

$$\nabla^2\overline{A}-k_0^2\overline{A}=-\mu_s\overline{J}. \tag{14}$$

The subscript "s" in eqs. (12), (13) and (14) is the spin mode's equivalent media parameter. Eqs. (13) and (14) are the spin equations and are different from Helmholtz eqs. (10) and (11).

1.3 Solution of spin equation, electric mode and magnetic mode

Now, let us solve eqs. (13) and (14) in spherical coordinates shown in Figure 2. Because EM field distribution is assumed to be rotationally symmetric, eq. (14) can be separated into three component equations as

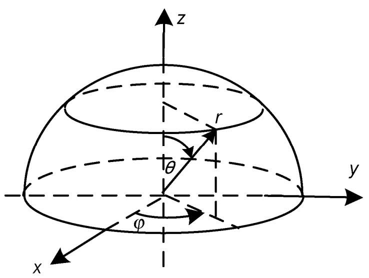

Figure 2 Spherical coordinate system.

$$\nabla^2 A_r-\frac{2}{r^2}\left(A_r+\frac{A_\theta}{\mathrm{tg}\theta}+\frac{\partial A_\theta}{\partial\theta}\right)-k_0^2A_r=-\mu_sJ_r, \tag{15}$$

$$\nabla^2 A_\theta+\frac{1}{r^2}\left(2\frac{\partial A_r}{\partial\theta}-\frac{A_\theta}{\sin^2\theta}\right)-k_0^2A_\theta=-\mu_sJ_\theta, \tag{16}$$

$$\nabla^2 A_\varphi-\frac{A_\varphi}{r^2\sin^2\theta}-k_0^2A_\varphi=-\mu_sJ_\varphi. \tag{17}$$

This indicates that A_φ in eq. (17) is independent of A_r and A_θ in eqs. (15) and (16). Therefore, we can obtain two orthogonal solutions. One is the electric mode, derived from ϕ, A_r and A_θ. The other is the magnetic mode, derived from A_φ.

(i) Spin electric mode. We defined scalar potation, ϕ as

$$\phi(\overline{r})=\phi_0R(\sigma)\Theta(\theta)\Phi(\varphi), \tag{18}$$

and the dipole source ρ as

$$\rho(\overline{r})=\frac{4\varepsilon k_0^2\phi(\overline{r})}{\sigma}, \tag{19}$$

where $\sigma=k_0r$ is the normalized radial component. After substituting them into eq. (13), we have

$$\begin{aligned}\frac{\nabla^2\phi}{\phi_0R\Theta\Phi}-k_0^2&=\frac{1}{R}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)\\&+\frac{1}{\Theta}\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)\\&+\frac{1}{\Phi}\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2}-k_0^2=-\frac{4\varepsilon k_0^2}{\varepsilon_s\sigma}.\end{aligned} \tag{20}$$

Because EM field is rotationally symmetric, we can obtain $\Phi(\varphi)=1$. $\Theta(\theta)$ takes the form $\Theta(\theta)=\cos\theta$, which satisfies $\frac{1}{\Theta}\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)=-\frac{2}{r^2}$. We then substituted Φ and Θ into eq. (20). After rearrangement, $R(\sigma)$ should satisfy

$$\frac{\partial^2R}{\partial r^2}+\frac{2}{r}\frac{\partial R}{\partial r}+\left[\frac{4\varepsilon k_0^2}{\varepsilon_s\sigma}-\frac{2}{r^2}-k_0^2\right]R=0. \tag{21}$$

The equivalent dielectric constant is

$$\varepsilon_s=\varepsilon, \tag{22}$$

and we obtain

$$\frac{\partial^2R}{\partial r^2}+\frac{2}{r}\frac{\partial R}{\partial r}+\left[\frac{4k_0}{r}-\frac{2}{r^2}-k_0^2\right]R=0. \tag{23}$$

This is a homogeneous equation, and its intrinsic solution is $R(r)=\sigma e^{-\sigma}$. Therefore, the expression of the potential function ϕ is

$$\phi(r,\theta)=\phi_0\sigma e^{-\sigma}\cos\theta, \tag{24}$$

where its normalized factor is

$$\phi_0=\frac{k_0 q}{4\pi\varepsilon}. \tag{25}$$

The charge density in eq. (19) becomes

$$\rho(\bar{r})=4\varepsilon k_0^2\phi_0 e^{-\sigma}\cos\theta. \tag{26}$$

The charge density distribution and electric potential density are shown in Figure 3.

Following the same methodology, we defined

$$J_r(\bar{r})=\frac{4k_0^2 A_r(\bar{r})}{\mu(\sigma+1)}, \tag{27}$$

$$J_\theta(\bar{r})=-\frac{4k_0^2 A_\theta(\bar{r})}{\mu(\sigma+1)}, \tag{28}$$

and the equivalent permeability of the mode

$$\mu_s=\mu. \tag{29}$$

Therefore, we can obtain the solution of eq. (15) as

$$A_r(r,\theta)=A_0(\sigma+1)e^{-\sigma}\cos\theta, \tag{30}$$

where current in radius direction is

$$\mu J_r(\bar{r})=4A_0k_0^2\cos\theta e^{-\sigma}. \tag{31}$$

The solution of eq. (16) is

$$A_\theta(\bar{r})=-A_0(\sigma+1)e^{-\sigma}\sin\theta, \tag{32}$$

where current in θ direction is

$$\mu J_\theta(\bar{r})=-4A_0k_0^2\sin\theta e^{-\sigma}, \tag{33}$$

and

$$A_0=\frac{\mu k_0 qc}{4\pi}. \tag{34}$$

Applying the Lorenz gauge (9), we have

$$A_0=\sqrt{\varepsilon_s\mu_s}\phi_0. \tag{35}$$

From eqs. (5) and (6), we can obtain all components of the electric mode EM field:

$$B_\varphi=A_0k_0\sigma e^{-\sigma}\sin\theta, \tag{36}$$

$$E_r=-2\omega_0A_0\cos\theta e^{-\sigma}, \tag{37}$$

$$E_\theta=\omega_0A_0(\sigma+2)e^{-\sigma}\sin\theta. \tag{38}$$

They satisfy Maxwell's equations, and the charge and current in eqs. (26), (31) and (33) meet the demands of the Continuity Law:

$$\nabla\cdot\bar{J}+\frac{\partial\rho}{\partial t}=0. \tag{39}$$

Figure 4 shows the plots of EM-field distribution in eqs. (36), (37) and (38)

(ii) Spin magnetic mode. The magnetic mode can be obtained from eq. (17). Let

$$J_\varphi(\bar{r})=\frac{4k_0^2A_\varphi(\bar{r})}{\mu\sigma}. \tag{40}$$

As we did for the electric mode for ϕ, the solution of eq. (17) is

$$A_\varphi(r,\theta)=A_0\sigma e^{-\sigma}\sin\theta, \tag{41}$$

and the current density is

$$\mu J_\varphi(\bar{r})=4A_0k_0^2\sin\theta e^{-\sigma}. \tag{42}$$

Their distributions are shown in Figure 5.

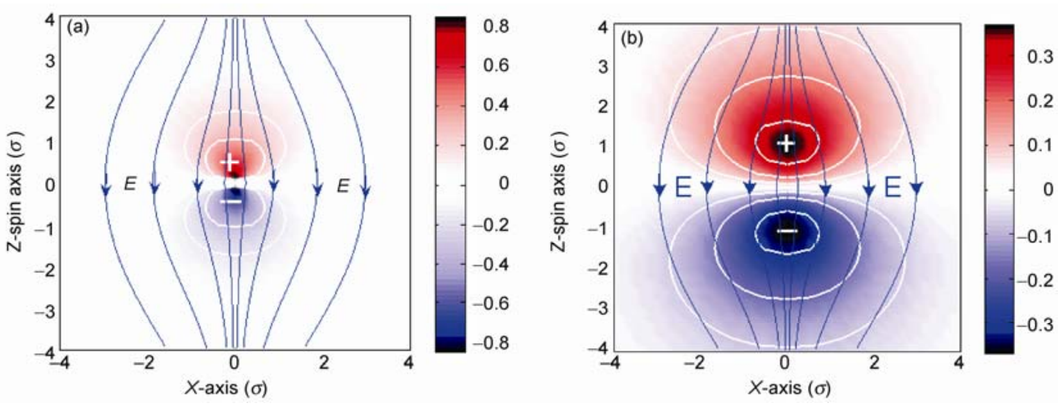

Figure 3 Charge density distribution (a) and the electric potential density (b). The blue lines represent the E field, and +/– denote polarity. The white lines represent the equipotential line. The figure goes through the z axis.

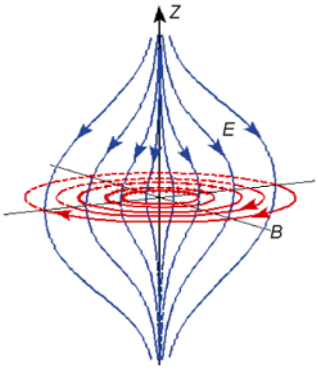

Figure 4 EM-field distribution of the electric mode.

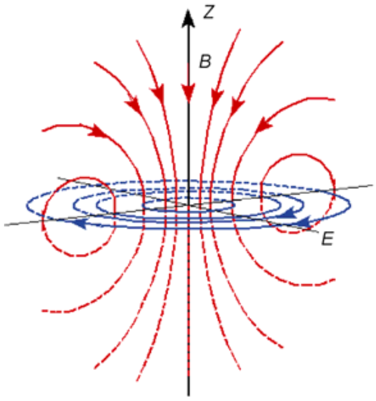

Figure 6 Field distribution of the magnetic mode.

Using eqs. (5) and (6), we can derive the expressions of the EM components of the magnetic mode as

$$E_\varphi = -\omega_0 A_0 \sigma e^{-\sigma} \sin\theta. \tag{43}$$

$$B_r = 2k_0 A_0 \cos\theta e^{-\sigma}, \tag{44}$$

$$B_\theta = k_0 A_0 (\sigma - 2) e^{-\sigma} \sin\theta. \tag{45}$$

The EM field is shown in Figure 6.

1.4 Characteristic parameters of the spin modes

The electric mode components are E_r, E_θ and B_φ, and the magnetic mode components are B_r, B_θ and E_φ. These modes are orthogonal to each other. They have three characteristic wave parameters: spin frequency, wave number and characteristic speed. The spin frequency ω_0 can be obtained first, and can be determined by excitation source.

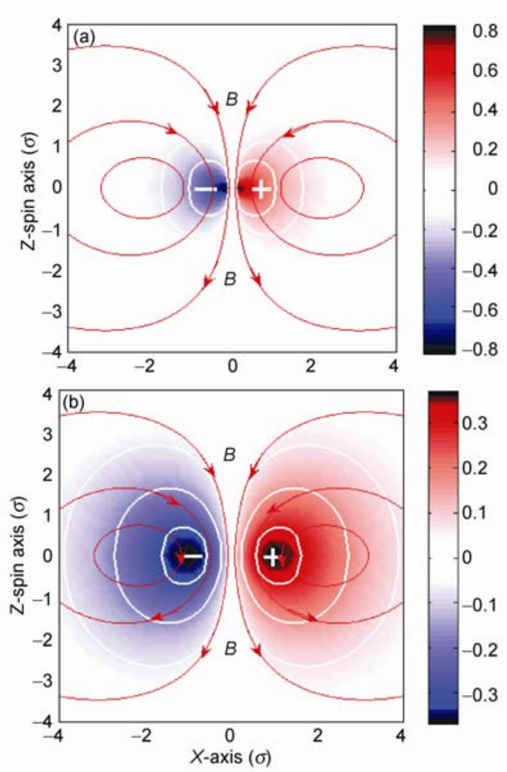

Figure 5 Current density J_φ (a) and potential A_φ (b) distributions. The +/– denote the direction of flow, the white lines represent the equipotential line, and the red lines denote the magnetic field.

(i) Wave number and intrinsic radius. The eigenvalue $k_0 = \omega_0 \sqrt{\varepsilon_s \mu_s}$ is a wave number in the radial direction. The characteristic radius can be obtained from this wave number:

$$r_0 = \frac{1}{k_0}. \tag{46}$$

This is the distance at which the exponential function decreases to 1/e. r_0 is one of most important parameters of spin mode. It determines the concentration of the EM energy. Also, we can derive the wave numbers in φ and θ directions at radius r_0: $k_\varphi = k_\theta = \frac{2\pi}{l_0} = \frac{2\pi}{2\pi r_0} = \frac{1}{r_0} = k_0$.

(ii) Characteristic speed. From eq. (12), $k_0 = \omega_0 \sqrt{\varepsilon_s \mu_s}$ we can derive the tangent speed at the characteristic radius, $v_0 = \frac{\omega_0}{k_0} = \frac{1}{\sqrt{\varepsilon_s \mu_s}} = c$, which is the characteristic speed of spin mode. It is determined by the EM equivalent parameters of the media. For a given media, r_0 is inversely proportional to ω_0. The higher ω_0 is, the smaller the r_0 is, and consequently, the higher the energy density.

2 Moment, energy and angular momentum of the spin modes

2.1 Spin electric mode

(i) Electric moment. The induced electric charge distribution of electric mode is expressed as $\rho(\bar{r}) = 4\varepsilon k_0^2 \phi_0 e^{-\sigma} \cos\theta$. We can calculate the electric moment as

$$P_z = \int_V \bar{r} \rho(\bar{r}) \cdot \hat{z} dV = \frac{q r_0}{2}. \tag{47}$$

This result shows that the electric mode is equivalent to a dipole with electric charge q separated by a distance $l=r_0/2$.

(ii) Energy. We can calculate the EM energy from eqs. (36)–(38) using an integral over the entire space.
The magnetic energy is

$$W_m = \frac{1}{2\mu_s}\int_V B_\varphi^2 \mathrm{d}V = \frac{q^2}{16\pi\varepsilon r_0},$$

and the electric energy is

$$W_e = \frac{\varepsilon_s}{2}\int_V \left(E_r^2 + E_\theta^2\right)\mathrm{d}V = \frac{5q^2}{16\pi\varepsilon r_0}.$$

The total energy is

$$W = W_e + W_m = \frac{6q^2}{16\pi\varepsilon r_0}. \tag{48}$$

Also, we can calculate the energy from the electric charge and current as $W^q = \frac{1}{2}\int_V \phi q \mathrm{d}V = \frac{q^2}{16\pi\varepsilon r_0}$ and $W^j = \frac{1}{2}\int_V \left(A_r J_r + A_\theta J_\theta\right)\mathrm{d}V = \frac{5q^2}{16\pi\varepsilon r_0}$. The total energy is the same as that obtained from the EM field. We can see that the energy of electric mode is inversely proportional to r_0, and directly proportional to ω_0.

(iii) Angular momentum. The axis of the spin mode is the z axis, but the angular momentum vector is perpendicular to z axis. That is, the electric moment and angular momentum are vertical to each other. We can select any vector on the x-y plane, for example, the x axis, to calculate the angular momentum. The result is

$$\begin{aligned} L_x &= \varepsilon_s \int_V \bar{r} \times \left(\bar{E}\times\bar{B}\right)\cdot\hat{x}\mathrm{d}V \\ &= \varepsilon_s \int_V \bar{r}\times\left(E_r B_\varphi \hat{\theta}\right)\cdot\hat{x}\mathrm{d}V = \frac{q^2}{8\pi\varepsilon v_0} = \frac{2W_m}{\omega_0}, \end{aligned} \tag{49}$$

where W_m is the magnetic energy of the electric mode.

2.2 Spin magnetic mode

(i) Magnetic moment. The induced current of magnetic mode is $J_\varphi\left(\bar{r}\right) = A_0 k_0^2 \sin\theta \mathrm{e}^{-\sigma}/\mu$. Therefore, the magnetic moment in the z direction is

$$M_z = \frac{1}{2}\int_V \bar{r}\times\bar{J}_\varphi\cdot\hat{z}\mathrm{d}V = \frac{r_0 q v_0}{2}. \tag{50}$$

This result shows that the spin magnetic mode is equivalent to a magnetic dipole with a circulating current, qv_0, and a diameter, $r_0/2$.

(ii) Energy. The electric and magnetic energy can be calculated separately, using eqs. (36)–(38): $W_e = \frac{\varepsilon_s}{2}\int_V E_\varphi^2 \mathrm{d}V = \frac{q^2}{16\pi\varepsilon r_0}$, $W_m = \frac{1}{2\mu_s}\int_V \left(B_r^2 + B_\theta^2\right)\mathrm{d}V = \frac{q^2}{16\pi\varepsilon r_0}$, and total energy is $W = \frac{q^2}{8\pi\varepsilon r_0}$. It also can be derived from the induced current:

$$W^j = \frac{1}{2}\int_V A_\varphi J_\varphi \mathrm{d}V = \frac{q^2}{8\pi\varepsilon r_0}. \tag{51}$$

This is total energy, where electric energy is equal to magnetic one.

(iii) Angular momentum. We can calculate the angular momentum on the x axis as we did for the electric mode:

$$\begin{aligned} L_x &= \varepsilon_s \int_V \bar{r}\times\left(\bar{E}\times\bar{B}\right)\cdot\hat{x}\mathrm{d}V \\ &= \varepsilon_s\int_V \bar{r}\times\left(E_\varphi B_r\hat{\theta}\right)\cdot\hat{x}\mathrm{d}V = \frac{q^2}{8\pi\varepsilon v_0} = \frac{2W_e}{\omega_0} = \frac{W}{\omega_0}, \end{aligned} \tag{52}$$

where W is total energy of the mode.

3 The two expressions of electric mode

The electric mode can be expressed using +/– charges, and the magnetic mode can be expressed by a circulating current. This is same as traditional definition for dipoles, but they are not a duality. To get the duality expression, we must find the equivalent magnetic current loop of the electric mode, or +/– magnetic charge of the magnetic mode. We only discuss this procedure for the electric mode in this paper. A similar procedure can be applied to the magnetic mode.

3.1 Current distribution of the electric mode

Let us rewrite the current distribution in cylindrical coordinates from eqs. (31) and (33) in spherical coordinates:

$$\begin{aligned} J_\rho &= J_r\sin\theta + J_\theta\cos\theta \\ &= -\frac{4A_0k_0^2}{\mu}\mathrm{e}^{-\sigma}\sin\theta\cos\theta + \frac{4A_0k_0^2}{\mu}\mathrm{e}^{-\sigma}\sin\theta\cos\theta = 0, \end{aligned} \tag{53}$$

$$\begin{aligned} J_z &= J_r\cos\theta - J_\theta\sin\theta \\ &= -\frac{4A_0k_0^2}{\mu}\mathrm{e}^{-\sigma}\cos^2\theta - \frac{4A_0k_0^2}{\mu}\mathrm{e}^{-\sigma}\sin^2\theta = -\frac{4A_0k_0^2}{\mu}\mathrm{e}^{-\sigma}. \end{aligned} \tag{54}$$

In Figure 7, it is shown that the current flows from positive charge to negative charge along the z axis. The current is not in a closed loop. However, the current is stably maintained, because it is an induced current caused by the spin of charge. Also, they satisfy the Continuity Law (eq. (39)).

3.2 Dual equations of the electric mode

By defining the equivalent displacement current: $\frac{\partial \bar{D}^*}{\partial t} =$

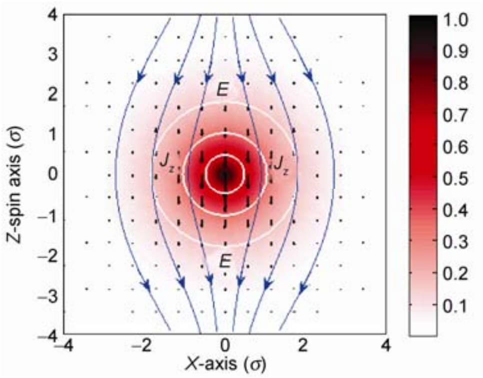

Figure 7 Longitudinal current J_z (black arrow) and electric field (blue line) distributions of the electric mode.

$\bar{J}+\dfrac{\partial \bar{D}}{\partial t}$, the curl eq. (2) becomes

$$\nabla\times\bar{H}=\frac{\partial \bar{D}^*}{\partial t},\tag{55}$$

where the superscript "*" denotes a magnetic current expression. We rewrite eq. (55) as

$$\bar{E}^*=\frac{1}{\varepsilon_s}\int_t \bar{J}\mathrm{d}t+\bar{E}.\tag{56}$$

We substitute it into the curl equation $\nabla\times\bar{E}=-\dfrac{\partial\bar{B}}{\partial t}$, and we can get

$$\nabla\times\bar{E}^*=-\frac{\partial\bar{B}}{\partial t}+\frac{1}{\varepsilon_s}\int_t\nabla\times\bar{J}\mathrm{d}t.\tag{57}$$

By defining the magnetic current as

$$\bar{J}^*=-\frac{1}{\varepsilon_s}\int_t\nabla\times\bar{J}\mathrm{d}t,\tag{58}$$

the curl eq. (57) becomes

$$\nabla\times\bar{E}^*=-\frac{\partial\bar{B}}{\partial t}-\bar{J}^*.\tag{59}$$

In this case, the magnetic field of the mode is unchanged, but the electric field is changed (as shown in eq. (57)), and we have

$$\nabla\cdot\bar{E}^*=\nabla\cdot\bar{E}+\frac{1}{\varepsilon_s}\int_t\nabla\cdot\bar{J}\mathrm{d}t$$
$$=\frac{\rho}{\varepsilon_s}+\frac{1}{\varepsilon_s}\int_t\nabla\cdot\bar{J}\mathrm{d}t=\frac{1}{\varepsilon_s}\int_t\left(\nabla\cdot\bar{J}+\frac{\partial\rho}{\partial t}\right)\mathrm{d}t=0.$$

Therefore,

$$\nabla\cdot\bar{E}^*=0.\tag{60}$$

This set of eqs. (55), (59) and (60), is the dual form of eqs. (1)–(3).

3.3 Electric mode expressed by the magnetic current

(i) Field structure. Substituting eqs. (31) and (33) into eq. (58), we can obtain the equivalent magnetic current as

$$\bar{J}^*=-\frac{1}{\varepsilon}\int_t\nabla\times\bar{J}\mathrm{d}t=-\frac{4A_0k_0^3}{\omega_0\varepsilon\mu}\sin\theta\mathrm{e}^{-\sigma}=-4\omega_0k_0A_0\sin\theta\mathrm{e}^{-\sigma}.$$

This is the duality form of the current in the magnetic mode expressed by eq. (42). Using the fact that the magnetic field of the electric mode is unchanged and eq. (56), the electric field can be simply derived:

$$E_r=2\omega_0A_0\cos\theta\mathrm{e}^{-\sigma},\tag{61}$$

$$E_\theta=\omega_0A_0(\sigma-2)\mathrm{e}^{-\sigma}\sin\theta,\tag{62}$$

$$B_\varphi=k_0A_0\sigma\mathrm{e}^{-\sigma}\sin\theta.$$

The electromagnetic structure of the electric mode is shown in Figure 8. It is the duality form of the magnetic mode (as shown in Figure 6).

(ii) Energy. The electric and magnetic energy calculated using magnetic current are $W_e=\dfrac{\varepsilon_s}{2}\displaystyle\int_V E_\varphi^2\mathrm{d}V=\dfrac{q^2}{16\pi\varepsilon r_0}$ and $W_m=\dfrac{1}{2\mu_s}\displaystyle\int_V\left(B_r^2+B_\theta^2\right)\mathrm{d}V=\dfrac{q^2}{16\pi\varepsilon r_0}$, respectively. The total energy is

$$W^*=\frac{1}{2}\int_V A_\varphi^*J_\varphi^*\mathrm{d}V=\frac{q^2}{8\pi\varepsilon r_0}.\tag{63}$$

The electric and magnetic energies are equal in this form of the electric mode. The total energy is the same as that of the magnetic mode in eq. (51).

(iii) Electric moment.

$$P_z=\frac{\varepsilon_s}{2}\int_V\bar{r}\times\bar{J}_\varphi^*\cdot\hat{z}\mathrm{d}V$$
$$=\frac{\varepsilon_s}{2}\omega_0A_0\int_V r\mathrm{e}^{-2\sigma}\sin^2\theta\mathrm{d}V=\frac{r_0q}{2}.\tag{64}$$

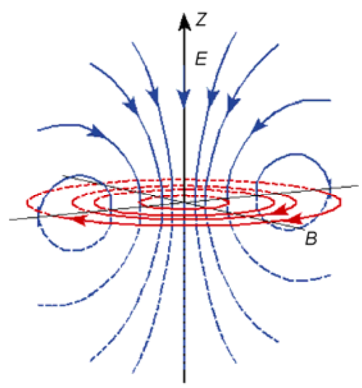

Figure 8 The electric mode expressed by a magnetic current.

This result shows that the electric mode has a torque with charge, q, separated by a distance, $l=r_0/2$. This result is equal to the previous one, eq. (47).

(iv) Angular momentum. The angular momentum value is the same as that from eq. (49), but the sign is different. That is,

$$L_x = \varepsilon_s \int_V \bar{r} \times \left(\bar{E} \times \bar{B}\right) \cdot \hat{x} \mathrm{d}V$$
$$= \varepsilon_s \int_V \bar{r} \times \left(E_r B_\varphi \hat{\theta}\right) \cdot \hat{x} \mathrm{d}V = \frac{q^2}{8\pi\varepsilon v_0} = \frac{W}{\omega_0}. \quad (65)$$

3.4 Divergent and non-divergent modes

The current along the z direction of the electric mode can be divided into two parts, one is the non-divergent ($\nabla \cdot = 0$) current, which represents a rotational field; and the other is the irrotaional ($\nabla \times = 0$) current which represents divergent field. The electric mode expressed by a pair of charge has both divergent and rotational field, and is a divergent mode, i.e. $\nabla \cdot \bar{E} = \dfrac{\rho}{\varepsilon}$. The divergent current satisfies the Continuity Law (eq. (39)). The vector potential satisfies Lorentz gauge. The electric energy is larger than magnetic energy in this mode.

The electric spin mode represented by magnetic current is a non-divergent mode. Non-divergent current satisfies eq. (58). After integration, the equation becomes

$$\nabla \times \bar{J} + \varepsilon \frac{\partial \bar{J}^*}{\partial t} = 0. \quad (66)$$

This shows the continuity between this part of the current and the magnetic current. In this case, there is no charge in the mode, and $\nabla \cdot \bar{E}^* = 0$. The electric mode expressed using the magnetic current is a kind of non-divergent mode with equal electric and magnetic energies.

The magnetic mode expressed using current is a non-divergence mode. Also, we can obtain the divergence mode expressed by magnetic charge using a similar process. In this case, the magnetic energy is larger than electric energy.

4 Further discussion on spin fields

4.1 Electric and magnetic curl fields and the resonant mode

The curl field is a basic physical phenomenon. Surrounding a wire carrying current, there is magnetic curl field, which decays along the radial direction. Maxwell proposed the concept of a displacement current and deduced the curl eq. (2) from Ampere's Law. Using the concept of the electric curl field, he also derived a curl eq. (1) from Faraday's Law [19]. An electric curl field is induced in a loop when the magnetic field passing through the loop is varying in time.

In metal or dielectric waveguide or cavity, the electric or magnetic curl fields will appear in resonant modes. The amplitude of the electric and magnetic field (including curl field) is harmonically time varying. The electric and magnetic fields reach their maximal values alternately. To maintain these modes, there must be current, charges or dipoles on the boundary of cavity that vary harmonically too. The mode in the spherical cavity in [20] has a similar field structure to that in spin mode described in this paper. The difference between them is the field distribution along radial direction. Mathematically, the mode in a cavity is a solution of wave equation, and the electromagnetic fields in cavity take the form of a standing wave.

The amplitude of the field of the spin mode is not time varying, and the spin of the mode maintains it. It is similar to a tornado or typhoon in the nature. It is a type of resonant mode as well, where electric or magnetic curl fields exist.

4.2 The source of spin electromagnetic field

Traditionally we use a δ-function or boundary condition to express the dipole source in the wave equation [21]. In other words, the source is separate from field in the equation. This is a very efficient mathematical tool and is widely used to solve EM problems. However, in some cases, especially that of free space, the obtained solution is non-physical and a singularity problem must be solved.

The concept of distributed sources or mode sources was adopted in this paper. There are relationships between potentials and sources. These are expressed by eqs. (19), (27), (28) and (40). Mathematically, these relations are used for solving eigenequations. There is no singularity problem, because the potential and sources are in the same space. Physically, the potentials and the sources are in one level, and the electromagnetic fields are in another. Physical quantities in either level can describe both the spin field and dipole. We can derive the field from the potential and source, or we can derive the potential and source from the field. Also, the calculated potential and source energy are equal to the energy of the field, which is shown in eqs. (48), (51) and (63). The expressions in these two levels are complementary.

There are two kinds of sources. One type is an entity such as charge or current; the other is displacement current or magnetic current. They correspond to the divergent and non-divergent modes. Because the Lorentz force acts on charges or currents, an interaction exists between the field and the divergent mode, but not non-divergent mode.

The current and charge on the wall of cavity and the displacement current of resonant mode can be used as analogue of a distributed source of a spin mode. We can excite the mode by electric or magnetic coupling. Therefore, we need not construct such a distributed source in space. In this way, we can describe the interaction between the spin mode and external field or source.

4.3 Localization and quantization of spin modes

A feature of the localization of a spin mode is its intrinsic radius. The radius is inversely proportional to its frequency, which is shown in eq. (46). The energy of the mode varies inversely with radius, which is shown in eqs. (48) and (51). The energy density of the mode is inversely proportional to the fourth power of the radius. At high frequencies, the energy is dense, the area is small, and the particle-like properties are more pronounced. By contrast, at low frequencies, wave properties are more evident. However, this depends on the means and scale of measurements used.

There are many monographs and text books discussing the quantization of the EM field [22,23], in which the wave equation was adopted. To express the EM field using energy, momentum and angular momentum is one of main objectives of quantization.

The spin EM field proposed in this paper is quantized. The energy and the frequency are directly proportional. The angular momentum which is shown in eqs. (49), (52) and (65) is independent of frequency. The undetermined constant in the amplitude, which is shown in eqs. (25) and (34), is the charge, q. It can be determined by driving source of the mode. If we choose the media parameters to be ε_0 and μ_0 in vacuum, introduce the fine structure constant α and the charge of electron e, let $q=\frac{e}{\sqrt{\alpha}}$, and then the angular momentum becomes $L_x=\frac{q^2}{8\pi\varepsilon v_0}=\frac{e^2}{8\pi\varepsilon_0\alpha c}=\frac{\hbar}{2}$, where $\hbar$ is Plank's constant. The energy of non-divergent mode in eqs. (51) and (63) becomes $W=\frac{\omega_0\hbar}{2}$. The energy of the divergent mode in eq. (48) is $W=\frac{3\omega_0\hbar}{2}$.

Also, the moment and angular momentum are perpendicular to each other. This shows the relationship between the electric dynamics and the mechanics.

5 Conclusions

The traditional method of solving Maxwell equations was based on this hypothesis: the magnitude of the EM field has harmonic time variation. Consequently, the wave equation was derived, and is used universally. Traditional EM theory is a macroscopic theory, which cannot be used in the microscopic world. However, there is no definite boundary between the macroscopic and microscopic worlds. Traditionally, it is assumed to be the atomic scale [24]. Rather than using a physical dimension, rotation is a more proper criterion in my opinion. The traditional EM theory is not suitable for objects with spin.

This paper proposed a different approach to solving Maxwell's equations. The starting point of the new method is the EM field spin. From this, we derived the spin equations and their intrinsic solutions in free space. The solution of the traditional wave equation is a linearly moving EM field. This is the first difference between the two methods. The second difference is that the solution of the spin equation is localized, but the solution of the wave equation is global. It is difficult to quantize the EM field for global solutions if no special method or hypothesis is used. However, the quantization of a localized field is natural. The third difference between the two methods is the handling of the source. The solution of the wave equation treats the source as a δ function. However, a distributed source is used in the spin EM field; the source and field coexist and are complementary.

Two types of solutions are appropriate for the two different moving states of EM fields.

Indeed, the spin equation introduced in this paper is very closely related to Schrödinger's equation in quantum mechanics. Eq. (23), which determines the scalar potential ϕ, can be changed into Schrödinger's equation if the variables of quantum mechanics are used. The solutions of Schrödinger's equation are localized but not rotating. The solutions of the spin equation are localized and rotating. However, it requires a vectorial potential and related equations in addition to the scalar potential.

The dipole is ubiquitous in the natural world. The characteristics and parameters of a dipole are revealed by the reaction between the dipole and EM field and can be determined through measurements. However, the structure of a dipole cannot be determined from its parameters, except schematically as shown in Figure 1. In this paper, we derived the EM field structure of a spin mode. From this EM field, we demonstrated all of the apparent parameters of a dipole. They are in accordance with the previous results from experiments. Therefore, we believe that the spin EM field can be used as a proper representation of a dipole.

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